IV. Conclusions

The transverse resonance approach associated with the Runge–Kutta numerical integration technique has been employed to calculate the propagation characteristics of the dominant mode in single-V-groove guide. Numerical results for the guide wavelength of the dominant mode in a single-V-groove guide have been obtained and shown to agree well with available results in the literature. The analysis of a double-V-groove guide has also been performed. Although the method described in this paper is applied to the analysis of the V-shaped groove guide, the analysis can be extended to quite general shapes such as semicircular grooves, trapezoidal grooves, and closed V-groove guides.

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Correct Determination of TE and TM Cutoff Wavenumbers in Transmission Lines with Circular Outer Conductors and Eccentric Circular Inner Conductors

Liyang Zhang, Jingjun Zhang, and Wenbing Wang

Abstract—**The cutoff wavenumbers of TE and TM modes (higher order modes) in transmission lines with circular outer conductors and eccentric circular inner conductors are carefully evaluated. The correctness of Kuttler's bounds is confirmed and the reason why some of the values obtained lie outside the bounds and some of the modes could not be found in Vishen's paper is given. A reliable technique for accurately determining the roots of an analytical function is proposed for finding cutoff wavenumbers in such a way as to avoid missing any modes.**

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I. Introduction

Calculation of the cutoff wavenumbers of TE and TM modes in transmission lines having a circular outer conductor and an eccentric circular inner conductor has been of great interest to many authors. By means of conformal mapping combined with intermediate methods for the lower bounds and the Rayleigh–Ritz method for the upper bounds, these cutoff wavenumbers were evaluated by Kuttler [1] for several different relative dimensions of the structure. A special analytical shape perturbation method [2] was developed by Roumeliotis *et al.* for treating small eccentricities. Vishen *et al.* [3] employed a method which may be conveniently used for evaluating the cutoff wavenumbers in the structure for large or small eccentricities and different radius ratios, and the examples in [1] were repeated. However, some of the results computed by Vishen obviously contradict Kuttler's bounds. As we can see from [4], some confusion still exists.

In this paper, the technique used in [3] is employed. Deficiencies of the formulation in [3] are pointed out, and by careful derivation, a new expression is obtained. Cutoff wavenumbers of the structure for all the cases considered in [3] are carefully evaluated. All of our results lie in the bounds reported by Kuttler and are quite close to the upper bounds. The reason why some of the modes could not be found and some of the values fall outside Kuttler's bounds in [3] is given.

Like the analytical technique used here, many other methods, among them the method of moments, also reduce cutoff wavenumbers to the zeros of an analytical function. Hence, correctly determining the zeros of an analytical function is a problem of general interest. Some iterative algorithms, e.g. Muller iteration, are frequently employed [5] for tackling the problem. To the best of the authors' knowledge, it is difficult to avoid missing roots by simply using such iterative algorithms, because one rarely knows how many zeros exist inside the given frequency band. In this paper, a new technique is developed on the basis of a combination of the contour integral method [6] with Muller iteration. The method exhibits accuracy and efficiency, as well as reliability.

II. Results and Discussion

We use the method employed in [3], i.e., separation of variables and the use of addition theorems for Bessel functions to satisfy the boundary condition at the outer circular conductor. This method, as pointed out by certain authors [4], is not new and can be found in many papers. However, with the matrix elements expressed in ratio forms of Bessel functions, the formulas presented in [3] have deficiencies which may sometimes result in mistakes. By a careful derivation (see the Appendix) we obtain, for TE modes,

$$\det\left[P_{nm}(k)\right]=0 \tag{1}$$

where the elements of the determinant are given by

$$P_{nm}(k)=\left[J_n'(kb)Y_m'(ka)-Y_n'(kb)J_m'(ka)\right]\cdot\left[J_{n-m}(kd)+(-1)^l J_{n+m}(kd)\right] \tag{2}$$

TABLE I
CUTOFF WAVENUMBERS FOR TE MODES

	Symmetric Modes				Antisymmetric Modes			
	Our Results	Vishen *et al.* [2]	Kuttler Bounds Lower	Kuttler Bounds Upper	Our Results	Vishen *et al.* [2]	Kuttler Bounds Lower	Kuttler Bounds Upper
$\eta = 2/3$ $d = 0.2$	1.32220	1.2522*	1.32027	1.32221	1.19175	1.1917	1.19001	1.19176
	2.4445	2.4365*	2.4408	2.4446	2.4304	2.4307*	2.4267	2.4305
	3.6217	3.6209	3.6157	3.6218	3.6202	3.6203	3.6142	3.6203
	4.7897	4.7897	4.7804	4.7901	4.7896	4.7896	4.7804	4.7899
	5.9378	5.9379	5.9218	5.9385	5.9379	5.9379	5.9231	5.9385
$\eta = 0.475$ $d = 0.315$	1.5158	1.4407*	1.5132	1.5159	1.3741	1.3740	1.3715	1.3741
	2.7343	2.7256	2.7270	2.7345	2.7196	2.7187	2.7125	2.7198
	3.9237	3.9240	3.8978	3.9248	3.9237	3.9244	3.9069	3.9247
	4.4035	—	4.342	4.407	5.0793	5.0796	5.041	5.083
	5.0801	5.0799	4.977	5.084	5.4232	5.3686	5.325	5.427
$\eta = 1/3$ $d = 2/9$	1.5806	1.5619*	1.5766	1.5807	1.5435	1.5435	1.5393	1.5436
	2.9065	2.9064	2.8968	2.9067	2.9064	2.9058	2.8966	2.9067
	4.1152	4.1152	4.0944	4.1161	4.1152	4.1152	4.0955	4.1161
	4.2342	4.4220*	4.2146	4.2356	5.1651	5.1606	5.131	5.167
	5.2673	5.2669	5.219	5.270	5.2758	5.2758	5.237	5.279
$\eta = 0.5$ $d = 0.2$	1.40792	1.3793*	1.40694	1.40793	1.35218	1.3522*	1.35114	1.35219
	2.6861	2.6849	2.6837	2.6862	2.6834	2.6838	2.6815	2.6840
	3.9295	3.9295	3.9247	3.9298	3.9295	3.9296	3.9247	3.9298
	5.0175	—	4.9937	5.0192	5.1131	5.1131	5.1036	5.1138
	5.1133	5.1131	5.1031	5.1139	5.8315	5.8106	5.793	5.834
$\eta = 0.25$ $d = 0.25$	1.6810	1.6650*	1.6768	1.6811	1.6489	1.6490	1.6446	1.6490
	2.9679	2.9678	2.9445	2.9684	2.9678	2.9667	2.9547	2.9682
	3.9861	—	3.939	3.988	4.1581	4.1579	4.120	4.160
	4.1650	4.1191	4.105	4.168	5.0581	5.0616*	4.978	5.060
	5.2946	5.2942	5.111	5.303	5.2964	5.2965	5.175	5.302
$\eta = 0.15875$ $d = 0.379$	1.7944	1.7769	1.7603	1.7948	1.7583	1.7584	1.7330	1.7584
	2.9992	2.9932	2.871	3.004	2.9879	2.9848	2.873	2.989
	3.7703	3.8632*	3.432	3.775	4.1614	4.1590	3.78	4.17
	4.1824	4.1808	3.76	4.21				

and for TM modes

$$\det[\boldsymbol{Q}_{nm}(k)] = 0 \tag{3}$$

with the elements of the determinant being

$$\boldsymbol{Q}_{nm}(k) = [J_n(kb)Y_m(ka) - Y_n(kb)J_m(ka)] \cdot \left[J_{n-m}(kd) + (-1)^l J_{n+m}(kd)\right]. \tag{4}$$

Here $\boldsymbol{P}$ and $\boldsymbol{Q}$, which stand for the determinants, are in boldface italics to distinguish them from the quantities in [3]. In both (2) and (4), $l = m$ for symmetric modes and $l = m + 1$ for antisymmetric modes.

It is obvious that both $\det[\boldsymbol{P}_{nm}(k)]$ and $\det[\boldsymbol{Q}_{nm}(k)]$ are analytical functions and they have no singularities in finite regions in the complex k plane. The cutoff wavenumbers are their zeros on the real axis. Many methods can be employed to determine the zeros of these functions.

In this paper, a new technique is developed for finding zeros of an analytical function with high accuracy without missing any roots. This is a problem of great importance, especially for determining the cutoff frequencies of a waveguide. The contour integral method, which has been widely used to compute the external resonant frequencies of scattering objects, is based on the residue theorem. With integrals along a closed contour, it can determine the number of zeros (with multiplicity counted) of an analytical function within a certain region (area inside the integral contour) in the complex plane, as well as their values with considerable accuracy. The method presented here, the contour integral method combined with Muller iteration, is a universal and powerful one for solving the problem. In our program, values given by the contour integral method, which usually have three or four significant decimal digits, are then used as the initial values for Muller iteration to obtain the final results, and higher precision is reached.

Since no poles of $\det[\boldsymbol{P}_{nm}(k)]$ and $\det[\boldsymbol{Q}_{nm}(k)]$ have to be taken into account, this algorithm is convenient to use. Provided the contour integral is along a rectangular contour enclosing the frequency band one is interested in, it can be numerically performed by Gaussian formulas with high accuracy. Although the procedure requires additional computation for contour integrals, it requires no more CPU time than that needed by simply using the Muller method, because iterations for each zero (usually four to six times in our procedure) are much less than those needed by using the Muller method only.

Results are normalized by letting the radius of the outer cylinder equal unity, i.e., $b = 1$. The zeros of (1) and (3) have been evaluated for several values of the eccentricity d and the radius ratio η for the first five TE and TM modes, both symmetric and antisymmetric. Numerical values accurate to the sixth decimal place have been obtained, with the order of the determinants up to 18.

Table I presents the symmetric and antisymmetric cutoff wavenumbers for TE modes, while the corresponding values for TM modes appear in Table II. The relative dimensions of the structure tabulated were selected to be the same as those in [1]

TABLE II
CUTOFF WAVENUMBERS FOR TM MODES

	Symmetric Modes				Antisymmetric Modes			
	Our Results	Vishen *et al.* [2]	Kuttler Bounds Lower	Upper	Our Results	Vishen *et al.* [2]	Kuttler Bounds Lowe	Upper
$\eta = 0.5$ $d = 0.1$	5.46953	5.4695	5.46911	5.47043	5.99176	5.9918	5.99121	5.99257
	6.47472	6.4747	6.47403	6.47547	6.92031	6.9203	6.91953	6.92102
	7.30617	7.3062	7.30527	7.30683	7.71232	7.7123	7.71130	7.71299
	7.86924	7.8692	7.86823	7.86982	8.4845	8.4845	8.4830	8.4852
	8.49647	8.4965	8.4950	8.4972	9.3564	9.3564	9.3542	9.3572
$\eta = 0.5$ $d = 0.2$	4.8106	4.8106	4.80935	4.81191	5.5114	5.5114	5.5098	5.5125
	6.1724	6.1724	6.1703	6.1735	6.7991	6.7991	6.7964	6.8002
	7.3945	7.3945	7.3907	7.3957	7.9607	7.9607	7.9559	7.9619
	8.4974	8.4974	8.4894	8.4991	9.0091	9.0091	8.9996	9.0106
	9.3409	9.3409	9.2694	9.3488	9.9556	9.9556	9.9316	9.9577
$\eta = 0.5$ $d = 0.3$	4.3071	4.3071	4.3042	4.3118	5.1224	5.1222	5.1179	5.1257
	5.8903	5.8903	5.8736	5.8944	6.6210	6.6210	6.5994	6.6251
	7.3197	7.3197	7.240	7.325	7.9910	7.9910	7.876	7.997
	8.2909	8.2909	8.081	8.316	9.1877	9.1877	8.829	9.210
	8.6388	8.6388	8.382	8.646	9.2676	9.2676	8.900	9.276
$\eta = 2/3$ $d = 0.2$	6.2399	6.2399	6.2379	6.2420	6.9683	6.9683	6.9654	6.9702
	7.6769	7.6769	7.6728	7.6787	8.3682	8.3682	8.3631	8.3700
	9.0439	9.0439	9.0323	9.0456	9.7053	9.7053	9.6922	9.7071
	10.3534	10.3536	10.318	10.356	10.9892	10.9892	10.947	10.992
	11.6134	11.6184*	11.539	11.616	12.2266	12.2266	12.128	12.229
$\eta = 0.25$ $d = 0.25$	3.4723	3.4723	3.4687	3.4752	4.2640	4.2640	4.2583	4.2680
	4.9221	4.9221	4.9110	4.9249	5.5393	5.5393	5.5239	5.5425
	5.9268	5.9268	5.893	5.930	6.6357	6.6357	6.582	6.641
	6.7154	6.7154	6.591	6.723	7.7135	7.7135	7.443	7.723
	6.7527	6.7527	6.622	6.767	7.7243	7.7243	7.488	7.735
$\eta = 0.25$ $d = 0.5$	2.9824	2.9824	2.887	2.996	4.0338	4.0338	3.858	4.043
	4.7868	4.7868	4.088	4.827	5.5432	5.5432	4.58	5.575
	5.8084	5.8084		5.877	6.9144	6.9144		6.992
	6.2439	6.2439		6.323	7.1560	7.1560		7.208
	7.5586	7.5592		7.735	8.1858	8.1858		8.355

and [3]. Both tables also list the results of Vishen *et al.* [3] and the lower and upper bounds provided by Kuttler [1] for purposes of direct comparison.

From the tables one can find that all of our results are within the bounds reported by Kuttler [1], and are very close to the upper bounds without any exception. Provided our results are correct, it can be found that for most cases Kuttler's upper bound is closer to the exact value and is good enough. Kuttler's upper and lower bounds are obtained independently by conformal mapping and the Rayleigh–Ritz method, respectively. Thus one can deduce that the Rayleigh–Ritz method is of high accuracy for the structure, even for high modes.

For TM modes, our results coincide with those of Vishen *et al.* [3] except for certain points, but for TE modes our results do not agree with those of [3] very well. In our opinion, there are two possible reasons why some of the roots (marked by a dash) are missing and some (marked by an asterisk) lie outside the bounds in [3]. The first is that the formulation in [3] has some deficiencies. Many extrinsic singularities (poles) exist in the expressions of $P_{mn}(k)$ and $Q_{mn}(k)$ in [3]. These poles do not indicate any characteristics of the structure but result from inappropriate mathematical derivation, and may sometimes lead to mistakes. For example, they may cause missing modes in some cases: when $d = 0$, if we select the radii a and b to let

$$J_m(k_1 a) = 0 \quad \text{and} \quad J_m(k_1 b) = 0$$

or

$$Y_m(k_2 a) = 0 \quad \text{and} \quad Y_m(k_2 b) = 0$$

it is obvious that k_1 and k_2 are cutoff wavenumbers of TM modes although they are not roots of $\det[Q_{mn}(k)]$. This means that under certain circumstances, the cutoff wavenumber (zero of $\det[P_{mn}(k)]$ or $\det[Q_{mn}(k)]$) may be canceled by the poles (see the Appendix).

The second reason is that the algorithm for finding zeros used in [3] is not good enough; hence some results are inaccurate and some modes are even missing. The functions get many discontinuities of the second kind along the real frequency axis, because $\det[P_{mn}(k)]$ and $\det[Q_{mn}(k)]$ have many poles. Hence, some algorithms, e.g. bisection, are unsuitable for searching for the zeros of these functions. On the other hand, by simply employing iterative algorithms, such as Muller iteration, it is also difficult to determine the number of zeros of the function within the given frequency band and to select good initial values. Nevertheless, without appropriate initial values and the number of zeros to find, iterative algorithms are somewhat blind and consequently apt to result in missing modes, as occurs in [3]. The low precision of certain values in [3], in the authors' opinion, should be ascribed to the poor quality of the computing program.

APPENDIX

Taking the TM mode (antisymmetric case) as an example, one gets from [3]

$$\psi(r,\theta)=\sum_{m=1}^{\infty}\left[A_mJ_m(kr)+B_mY_m(kr)\right]\sin(m\theta) \quad \text{(A1)}$$

and

$$A_mJ_m(ka)+B_mY_m(ka)=0. \quad \text{(A2)}$$

By the addition theorem for Bessel functions,

$$Z_m(kr)e^{jm\theta}=\sum_{p=-\infty}^{\infty}Z_{m+p}(kr')J_p(kd)e^{j(p+m)\theta'}. \quad \text{(A3)}$$

Thus we obtain the wave function on the outer conductor:

$$\psi(r,\theta)|_{r'=b}=\sum_{m=1}^{\infty}\left\{\sum_{p=-\infty}^{\infty}\left[A_mJ_{m+p}(kb)+B_mY_{m+p}(kb)\right]\cdot J_p(kd)\right\}\sin(m+p)\theta'. \quad \text{(A4)}$$

Let $n=m+p$, and change the order of summation in (A4):

$$\psi(r,\theta)|_{r'=b}=\sum_{n=1}^{\infty}\left\{\sum_{m=1}^{\infty}\left[A_mJ_n(kb)+B_mY_n(kb)\right]J_{n-m}(kd)-\left[A_mJ_{-n}(kb)+B_mY_{-n}(kb)\right]J_{-m-n}(kd)\right\}\cdot\sin(n\theta').$$

It is known that

$$Z_{-n}(x)=(-1)^nZ_n(x).$$

Hence

$$\psi(r,\theta)|_{r'=b}=\sum_{n=1}^{\infty}\left\{\sum_{m=1}^{\infty}\left[A_mJ_n(kb)+B_mY_n(kb)\right]\cdot\left[J_{n-m}(kd)+(-1)^{m+1}J_{n+m}(kd)\right]\right\}\sin(n\theta'). \quad \text{(A5)}$$

On the outer conductor, the Dirichlet boundary condition should be met, that is,

$$\psi(r,\theta)|_{r'=b}=0. \quad \text{(A6)}$$

From (A5), $\psi(r,\theta)$ is represented as a Fourier series of θ'. So we deduce that

$$\sum_{m=1}^{\infty}\left[A_mJ_n(kb)+B_mY_n(kb)\right]\cdot\left[J_{n-m}(kd)+(-1)^{m+1}J_{n+m}(kd)\right]=0, \qquad n=1,2,3,\cdots. \quad \text{(A7)}$$

And by (A2), we can define

$$C_m=\frac{A_m}{Y_m(ka)}=-\frac{B_m}{J_m(ka)}.$$

Note, if $C_m=0$, then $A_m=B_m=0$ can always hold, and if $C_m\neq 0$, we can deduce $A_m\neq 0$ or $B_m\neq 0$ (because $Y_m(ka)$ and $J_m(ka)$ can never have the same zeros). Hence (A7) is equivalent to the following equations:

$$\sum_{m=1}^{\infty}C_m\left[J_n(kb)Y_m(ka)-Y_n(kb)J_m(ka)\right]\cdot\left[J_{n-m}(kd)+(-1)^{m+1}J_{n+m}(kd)\right]=0, \qquad n=1,2,3\cdots \quad \text{(A8)}$$

written in matrix form, i.e.,

$$[\boldsymbol{Q}_{nm}(k)][C_m]=0$$

in which m and n indicate columns and rows, respectively, and

$$\boldsymbol{Q}_{nm}=\left[J_n(kb)Y_m(ka)-Y_n(kb)J_m(ka)\right]\cdot\left[J_{n-m}(kd)+(-1)^{m+1}J_{n+m}(kd)\right].$$

Suppose, for $k=k_{\mathrm{TM}}$ that makes $\det[\boldsymbol{Q}_{nm}(k)]$ vanish, i.e.,

$$\det\left[\boldsymbol{Q}_{nm}(k_{\mathrm{TM}})\right]=0.$$

Then of course one can get a nontrivial solution $[C_m]\neq 0$. So, $[A_m]\neq 0$ or $[B_m]\neq 0$, and the field can take a nontrivial solution. This means that k_{TM} represents a cutoff wavenumber of the waveguide. Otherwise, if

$$\det\left[\boldsymbol{Q}_{nm}(k_{\mathrm{TM}})\right]\neq 0$$

one readily deduces that $[C_m]$ can take a trivial solution only, and consequently $[A_m]$ and $[B_m]$ take a trivial solution simultaneously. This means that the field takes a trivial solution, so k_{TM} cannot be a cutoff wavenumber.

If one defines

$$C_m=J_m(ka)A_m=-Y_m(ka)B_m$$

expressions for $Q_{mn}(k)$ in [3] can be obtained. However, problems sometimes arise. Although $[C_m]$ has a trivial solution only, $[A_m]$ and $[B_m]$ can take a nontrivial solution, provided ka is a zero of $J_m(x)$ or $Y_m(x)$. This means that when the cutoff wavenumber coincides with a zero of $J_m(x)$ or $Y_m(x)$, it will probably be missing.

Introducing a Neumann boundary condition into the above procedure and taking derivatives with respect to r' on both sides of (A3), one can obtain expressions for $\boldsymbol{P}_{nm}(k)$. A discussion of TE modes is similar to the above.

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An Efficient Algorithm for Transmission Line Matrix Analysis of Electromagnetic Problems Using the Symmetrical Condensed Node

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Abstract—**The symmetrical condensed TLM node has been closely examined. An efficient algorithm has been developed from the results of this study which significantly improves the numerical efficiency of the node. Certain physical aspects of the symmetrical condensed node are also discussed.**

I. Introduction

The transmission line matrix (TLM) method has now been established, owing to the works of Johns [1] and Hoefer [2], as one of the most powerful time-domain solvers of electromagnetic problems [3], [4].

The symmetrical condensed node invented by Johns [5] has proved to be a particularly valuable tool in TLM analysis. Since it represents both the electric and the magnetic field at the same point in space, it is more attractive than the expanded node used in other TLM networks [6], in the Finite-Difference Time-Domain (FDTD) method [7], and in the spatial network method [8]. Besides, as a consequence of the simplicity of node topology, ambiguities of interfaces and boundaries are removed. The node has recently been extended to cover lossy media [9].

The disadvantage of the symmetrical condensed node is that no equivalent circuit can be drawn up to represent it. It is solely characterized by a scattering matrix. The user, therefore, has to perform a linear transformation using an 18×18 scattering matrix at each nodal point. This means that the numerical efficiency is inherently low. Further, the nature of this scattering matrix has rarely been discussed.

In this paper, we demonstrate that an efficient algorithm can be obtained for the symmetrical condensed node. Such an algorithm not only shortens the computation time but also helps to unlock the physics hidden behind the scattering matrix. We shall first discuss the case of the original node before moving on to the lossy node. Finally, numerical examples will be presented.

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II. The Symmetrical Condensed Node

The basic structure of the symmetrical condensed node as proposed by Johns is given in Fig. 1. It is connected to each of its six neighbors by a pair of transmission lines, carrying orthogonal polarizations. These lines are numbered 1 to 12. The node is also connected to six stubs, one for each field component. The three electric or permittivity stubs (numbered 13 to 15) are open-circuit, while the magnetic or permeability stubs (16 to 18) are short-circuit. Hence, each node receives 18 input impulses at each time step.

Scattering takes place at the center of the node. The 18 input impulses $\underline{V}^i$ are scattered to produce 18 output impulses $\underline{V}^s$ into the 18 ports:

$$\underline{V}^s = \boldsymbol{S}\cdot\underline{V}^i. \tag{1}$$

The scattering matrix, $\boldsymbol{S}$, has been derived by Johns from Maxwell's equations and is shown in Fig. 2. The elements of the matrix assume the following values:

$$a_{pq} = \frac{-Y_p}{2(4+Y_p)} + \frac{Z_q}{2(4+Z_q)}$$

$$b_p = e_p = \frac{4}{2(4+Y_p)}$$

$$c_{pq} = \frac{-Y_p}{2(4+Y_p)} - \frac{Z_q}{2(4+Z_q)}$$

$$d_q = i_q = \frac{4}{2(4+Z_q)}$$

$$f_q = Z_q d_q$$

$$g_p = Y_p b_p$$

$$h_p = \frac{(Y_p-4)}{(Y_p+4)}$$

$$j_q = \frac{(4-Z_q)}{(4+Z_q)}$$

where the subscripts $p, q = x$, y, or z. The subscript p is related to the associated permittivity stub of the port in question and q is related to the associated permeability stub (See Fig. 2 for the associations). For example,

$$S_{29} = c_{xy}.$$

Note that Y_p is the normalized characteristic admittance of the electric stub p, and Z_q is the normalized characteristic impedance of the magnetic stub q.

III. The Scattering Matrix

Although the scattering matrix appears to be very complicated, it possesses a high degree of symmetry. We have ex-